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COMMENT

Reply to comment 'Exact solution of an *N*-body problem in one dimension'

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Abstract. It is pointed out that apart from the classic Calogero problem, the *N*-body potential discussed in reference [1] of the preceding comment is the only one for which the *N*-body problem as given by equation (1) of the comment is analytically solvable for all partial waves. In that sense the *N*-body potential is indeed unique.

In the comment [1], Calogero has made two points.

(i) The bound-state spectrum obtained in [2] for the N-body problem experiencing the 'Coulomb' potential $v(r) = -\alpha'/r$ is a special case of the more general result valid for any 'radial' potential $v(r)$.

(ii) The claim $p'_j = p_{N+1-j}$ made in [2] concerning the scattering is incorrect where p_j, p'_j $(j, j' = 1, 2, \ldots, N)$ are the initial and final momenta, respectively.

Calogero is right about his point (ii) and it must be admitted that in [2] one had overlooked the fact that the phase shift η_p also depends on the quantum number k and hence cannot be taken out of the summation sign‡.

Calogero, however, has missed an important fact in his first point. Even though it is true that the energy spectra of the N-body Hamiltonian as given by equations (1*a*) and (1*b*) of [1] coincides (except for multiplicities) with the eigenvalues E of equation (6*a*) (or equation (6*b*) of [1]), it is worth pointing out that of all possible $v(r)$ the only two problems for which the entire spectrum can be written down analytically for all partial waves are when $v(r) = ar^2$ or $v(r) = -\alpha'/r$. The first case was discussed by Calogero in his classic 1969 paper [3], while the second case was discussed in [2]. As is well known, in these two cases, not only is the spectrum analytically solvable but there are also huge degeneracies in both the cases. One can of course also add a term $g'/\left(\sum_{j>k}(x_j - x_k)^2\right)$ in both the cases and the problem is still analytically solvable but the degeneracy gets lifted.

I might also add here that even in the case of N-anyons (in two dimensions), a class of exact solutions can only be obtained for all partial waves if the N-anyons are interacting either by the two-body harmonic interaction or by the N-body interaction [4] of the type given in [2]. This is also the case for N-body problems in arbitrary number of dimensions [5] as can easily be seen by going over to hyperspherical coordinates.

In conclusion, apart from the classic Calogero problem [3], the N-body potential as discussed in [2] is the only other example for which the N-body problem as given by

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‡ This fact was also pointed out to me recently by N Gurappa, C N Kumar and P K Panigrahi of the University of Hyderabad, India.

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equation (1) of [1] can be solved analytically for all partial waves and in that sense the example discussed in [2] is indeed unique.

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